On the 150th Birthday of Max Planck:

On Honesty Towards Nature[1]

by Caroline Hartmann

The great physicist Max Planck would have been 150 years old on April 23, 2008. In discovering the correct equation for the description of heat radiation (the famous "Radiation formula"), he blazed a new trail for physics. His formula contains the postulation $E = h\nu$, that is, that energy is available in so-called quanta. It is thanks to Planck's integrity and strength of character that this true explanation of heat radiation prevailed, because the discussion at that time was anything but honest, above all when one considered the methods of a Niels Bohr. For, the Copenhagen interpretation, the uncertainty principle, and quantum mechanics are pure mathematical-statistical "interpretations." Almost all scientists at the time fell in with the mathematical euphoria, without exact knowledge of the true physical processes. First one had to have a "System," then came the discoveries.

Already as a young physicist Max Planck had found that the world of established, so-called classical, physics, as represented by famous big-name professors like Robert Clausius, Hermann von Helmholtz, and others, suffered from some problems with the understanding of various natural phenomena, and above all with the acceptance of new and far-reaching ideas. In his prize-winning work of 1887, "Das Prinzip der Erhaltung der Energie" (The Principle of the Conservation of Energy), submitted for a contest sponsored by the Göttingen philosophy department, Planck had mentioned the work of Robert Mayer, the discoverer of the mechanical equivalent of heat, and especially his explanation of the phenomenon of heat. *Heat* is usually falsely explained as accelerated molecular motion of matter or bodies, that is, heat energy is a pure mechanical kinetic energy. Robert Mayer, who grappled intensively with the phenomenon of *vis (kraft)* had expressly noted in his discovery that heat, which is a kind of *vis* (today one says *energy*), is equivalent to

[1] *From an article appearing in the German-language newspaper* Neue Solidaritaet (No. 18/2008), *translated by Laurence Hecht.*

the mechanical motive force, however, that this “heat energy” (*Wärmekraft*) ought not be expressly reduced to the increased motion of the smallest existing part of matter (see also, “Was ist Wärme? Oder: warum die Natur keine Disco ist,” (What is Heat? Or why Nature Is not a Disco) in *Neue Solidarität* Nos. 17 and 18, 2006).

A purely “mechanistic” explanation of heat would be impermissible and unfounded, according to Robert Mayer. That is also the point that Planck stressed throughout his life. Mayer’s discovery pointed to concepts far into the future of this new field of physics, *thermodynamics,* but the then leading figures in physics, Hermann Helmholtz and Robert Clausius, reduced them to a purely “mechanistic” interpretation of heat phenomena and simply imported the already known laws of mechanics into the molecular domain. Thus began the dilemma over the fundamental understanding of Nature, which would break out anew after Plank’s discovery.

Max Planck was born in Kiel on April 23, 1858. By 1867, the family had relocated to Munich, where the father was appointed professor of law at the university. His mother came from a family of ministers. His great-great grandfather Gottlieb Jakob Planck (1751-1833), Professor of Theology at Göttingen University, belonged to the circle of Abraham Gotthelf Kästner who brought Benjamin Franklin to Göttingen in 1766, and published the first translation of Leibniz’s answer to John Locke’s misanthropic theory, the *New Essays on Human Understanding.* The thinking of that great philosopher and mathematician also shaped Max Planck himself.

After graduation from high school, Planck studied in Munich for three years, and another year in Berlin under Helmholtz and Kirchoff. Concerning Helmholtz he reported:

“Sadly I must admit that his lectures brought me no appreciable advantage. Helmholtz obviously never prepared properly; he spoke only haltingly, picking out the needed data from a little notebook, besides consistently miscalculating at the blackboard, and we had the feeling that he was at least as bored by his presentation as we were.”

In 1878, the just 20-year old Planck wrote his doctoral thesis in less than four months. And after intensive study of the vastly different works on thermodynamics, for example that of Robert Clausius and Robert Mayer, he wrote the aforementioned essay, "The Principle of the Conservation of Energy," where he challenged the narrowly conceived notion of heat based purely on motion. Planck was firmly convinced that Nature and the universe acted according to determined rules, which are lawfully knowable to man, not by the accidental whims of statistics and probability.

After his first years at the University of Kiel, in 1889 Planck was asked by the Berlin Philosophical Faculty to become the successor to Gustav Kirchoff (1824-1887) in the post of theoretical physics. In 1894, he was nominated to the Prussian Academy of Sciences. In the following year he plunged into research aimed at widening the reach of thermodynamics. He subjected to fundamental questioning the mechanistic interpretation of heat advocated by Herman Helmholtz who, incidentally, in his 1847 writing "Über die Erhaltung der Kraft" (On the Conservation of Force), never mentioned Mayer's priority of publication of the discovery of the heat equivalent. Planck wrote:

"It is worthy of note, that with the discovery of the mechanical equivalent of heat and the development of the general principle of the conservation of energy, the belief that all natural phenomena consist in motion, went hand in hand and became virtually identical with it. Yet strictly speaking, the principle of the conservation of energy expresses no more than the convertibility of particular natural forces into one another according to fixed relationships, but sheds absolutely no light on the way in which this conversion takes place. It is in no way permissible to deduce from the applicability of the principle of conservation of energy, the necessity of the mechanical view of nature, while conversely, the principle of conservation of energy always emerges as a necessary result of the mechanical view, at least when one proceeds from central forces."

Max Planck was the sort of person who could never attribute an evil motive to another, so long as the contrary was not proven. He was, however, aware of the abstruse arguments of a Helmholtz or Lord Kelvin, who, from precisely this mechanistic world view, had taken for granted the ultimate "heat death" of the universe as a consequence of entropy. Planck was also well aware of the not very scientific habit of Helmholtz of routinely selling the works and ideas of others as his own. Throughout his life, Planck fought the conclusion which Robert Clausius had drawn from this overly narrow view of natural phenomena— namely the theorem that there exists a continual increase in universal entropy (known as the Second Law of Thermodynamics):

"This hypothesis demands special comment. For, it should not only be expressed by this hypothesis that heat does not flow directly from a colder into a warmer body, but also that it is in no way whatsoever possible, to get heat out of a colder body into a warmer one, without some alteration in nature remaining behind as compensation."

Such an instance, namely "the process of heat conduction being in no way whatever completely reversible," Planck accepts as a matter of course; today it has become accepted under the concept of "irreversibility." However, a fundamental difference is lurking here; the failure to recognize it has had a negative impact on the entire further development of the understanding of heat phenomena. Planck wrote:

"However, the error committed by an overly narrow interpretation of Clausius's theorem, and which I have fought against tirelessly for my entire life, is, it seems, not to be eradicated. For, up to the present day, instead of the above definition of irreversibility, I have encountered the following: `An irreversible process is one that cannot run in the reverse direction.' That is not adequate. For, at the outset, it is well conceivable that a process which cannot proceed in the reverse direction, by some means or another can be made fully reversible."

The more detailed investigation of heat, alongside the understanding that all radiation derives from the same process, and the various types are differentiated only by their frequency—postulated by Ampère, and then formulated as a law by Gustav Kirchoff— should have brought this mistaken and overly narrow conception into focus again. Unexpected and phenomenal discoveries in the investigation of the spectra of radiating bodies pointed to a certain constant regularity in the microscopic realm of the atomic construction of matter.

What Is Heat Radiation?

At the beginning of the 19th Century, the prevailing view still was that the various types of radiation were completely different as regards their refrangibility and other properties. There was visible light, which could be seen coming from the Sun or other glowing bodies; pure heat rays, which could be felt emanating from heated bodies, for example a hot iron bar, and the chemically active rays (ultraviolet rays). Practically, in order to account for the natural phenomena, one started out from the human sensory impressions. However, to be able to find the real processes at play, one must look beyond these phenomena. That was done by the French physicist André-Marie Ampère, who asserted: One and the same process must lie behind all the various types of radiation. For, light rays must be nothing other than visible heat rays, and the chemically active rays just heat rays of a higher frequency. That means that the types of radiation are distinguished only by their wavelength (frequency $\nu = 1/\lambda$), and one can arrange them into a continuous spectrum.

Our eyes, says Ampère, can only perceive a specific region of the spectrum as "light," while they do not react to rays of other refrangibility. This insightful hypothesis emerged over time as the true one; however, it took a long time before it was proven that the radiation spectrum was actually continuous, i.e., that at every wavelength there existed a measurable radiation. Experimental physicists, including such investigators as Gustav Kirchoff, Robert Bunsen, Ernst Pringsheim, and Otto Lummer, concerned themselves with the trailblazing discoveries which ultimately led to Planck's discovery of the true law of radiation, and to a completely new understanding of physics.

With "Bunsen's Lamp" (today known as the Bunsen burner), these scientists examined the spectrum of all kinds of materials, and came upon a completely unexpected phenomenon, which Kirchoff described in his publication "Über das Verhältnis zwischen dem Emissions- und Absorptionsvermögen der Körper für Wärme und Licht" (On the Relationship between the Ability of Bodies to Emit and Absorb Heat and Light):

"If a definite body, a platinum wire, for example, is heated until it attains a certain temperature, it will emit—up to a certain temperature—only rays of wavelength greater than the visible rays. At a certain temperature, rays of infrared wavelength begin to appear; as the temperature rises higher and higher, rays of smaller and smaller wavelength are added, such that at each temperature rays of a corresponding wavelength appear, while the intensity of the rays of longer wavelength may grow.… It follows from this … that all bodies, when their temperature is gradually raised, begin to emit, at the same temperature, rays of the same wavelength, and thus begin to glow red at the same temperature, and at a higher temperature, they all begin to give off yellow rays, and so forth. The intensity of the rays of given wavelengths, which different bodies emit at the same temperature, can however be very different….”

How should this be explained? It can only have to do with the inner construction of the matter.

At the same time, a man by the name of Mendeleyev fought for his hypothesis in Russia, that there is a periodicity in the atomic weights of the elements. Amidst the general clutter of matter, he asserted that mass is not a simple linear function, but shows a harmonicity when the elements are arranged according to what we know today as Mendeleyev’s periodic table. By 1860, a few years before Mendeleyev’s great discovery, just 60 elements were known. The work of Kirchoff and Bunsen in corroborating Mendeleyev’s thesis was of fundamental significance, and it is not surprising that they discovered two new elements (cesium and rubidium) through spectral analysis of the mineral water from Bad Dürkheim.

To better investigate these phenomena, which appear repeatedly in the same way in all matter, Kirchoff conceived of the ideal possibility of collecting all the rays at the same time in a closed cavity (*Hohlraum*), a so-called “black body.” That could be, for example, a metal pipe, which is painted black to minimize the escape of radiation, and to thus obtain an equilibrium condition among the reflecting and refracting waves within the body. The pioneering discovery of the year 1900, which showed that the energy is always partitioned in exactly the same way among the different wavelengths, independently of the character of the material, was published by Lummer and Pringshiem in the *Proceedings of the German Physical Society* under the title “Über die Strahlung des schwarzen Körpers für lange Wellen’ (On the long wave radiation of black bodies). This characteristic energy distribution of the radiation was completely incomprehensible from the standpoint of the prevailing understanding of the wave behavior of light. Planck described it as follows:

"Imagine a body of water on which strong winds have generated high waves. After the wind stops, the waves will persist for some time and roam from shore to shore. However, they will experience a certain characteristic alteration. Especially as a result of their impact against the shore or other fixed objects, the kinetic energy of the longer, larger waves will be increasingly changed into the kinetic energy of shorter finer waves, and this process will persist until, finally, the waves become so small, and their motion so faint, as to become imperceptible. Hence, the well-known conversion of macroscopic into molecular motion, and ordered motion into unordered. For, in ordered motion, neighboring molecules share a common velocity, while in the disordered, each molecule possesses its own, peculiarly directed velocity.

"However, the process of splitting up (scattering) described here does not go on indefinitely, but finds a natural limit in the size of the atom. For the motion of a single atom, taken by itself, is always ordered, since the individual parts of an atom all move with the same common velocity. The larger the atom, the smaller can be the splitting up of the total kinetic energy. So far it is all perfectly clear, and the classical theory best corresponds with experiment.

"Now let us think of a completely analogous process—not with waves of water but of light and heat radiation—and assume, for example, that by provision for adequate reflection, the rays emitted by an intensely heated body would be collected within an enclosed cavity (*Hohlraum*), and constantly thrown back and forth between the reflecting walls of the cavity. Here also, a gradual transformation of the radiant energy from longer to shorter waves, from ordered to disordered, will take place; the longer, larger waves correspond to the infrared, the shorter, finer to the ultraviolet part of the spectrum. According to the classical theory, one would expect that the whole radiant energy finally ends up in the ultraviolet part of the spectrum, or, in other words, that the infrared and visible rays gradually disappear altogether, and are changed into the invisible ultraviolet rays which evince predominantly only chemical action.

" However, no trace of any such phenomenon can be found in Nature. In fact, the transformation sooner or later becomes completely determined, in a precisely detectable end result, and from thence the condition of the radiation remains stable in that respect."

(from the lecture “New Paths in Physical Knowledge,” delivered by Planck on October 15, 1913, on the acceptance of his Rectorship of the Friedrich Wilhelm University in Berlin).

These results gave evidence of a constant relationship, and Planck, firmly convinced that an explanation of fundamental processes in the universe could be found from these fixed natural constants, worked intensively for a solution:

“From the experimental measurements of the spectrum of heat radiation made by Lummer and Pringsheim at the government Physical-Technical Institute, my attention was directed to Kirchoff’s theorem, that in an evacuated cavity surrounded by perfectly reflecting walls and containing any emitting and absorbing body whatsoever, over time a condition is reached, in which all bodies take on the same temperature, and the radiation in all its properties, including the distribution of its spectral energy, depends not upon the character of the body, but only upon its temperature. This so-called normal energy distribution thus represents something absolute, and as the search for the absolute always seemed to me to be the most beautiful problem to research, this examination became my passion.”

Is Nature Based on Statistical Accidents?

The formula, which Planck ultimately discovered, implied the condition $E = hv$, which states that matter can only absorb energy in determined portions (*quanta*). Thus did the old debate, whether radiation consisted of waves or particles, blaze up again. Planck was somewhat shocked by the fireworks he had set off in physics, and had to assert that there were still too few facts, and also too few physicists who appreciated the necessity for an urgent reform of so-called “classical physics.” And facts could ultimately only be gotten by experiment:

“My futile attempts to incorporate the Quantum of Action into classical physics extended over a number of years, and cost me much work. Many colleagues saw in that a kind of tragedy. I am of another opinion. For the benefit that I got from such fundamental investigation was the more valuable. Now I knew for sure that the Quantum of Action played a very important role in physics, just as I had been inclined to assume from the start."

"However, precisely the existence of a kind of objective limit, as is represented by the elementary quantum of action, must be judged as evidence for the rule of a certain new kind of Lawfulness, which certainly cannot be ascribed to statistics. Clearly nothing was left but the admittedly very radical, but obvious, assumption, that the elementary concepts of classical physics no longer suffice in atomic physics."

Planck was already familiar with the attitude of people like Helmholtz and Clausius toward fundamental questions of physics, based as it was on vanity and the desire for fame. However, what now took place exceeded both "personal" craving for recognition and dogmatism; it was conscious sabotage of the search for truth. The Swedish Academy appealed to the authority of Hendrik Anton Lorentz, professor of theoretical physics at the University of Leyden, who was admired as one of the greatest physicists. He made clear at the start that Plank's formula lacked a "satisfactory theoretical basis," and he authored a demonstration that Planck's formula was not derivable from classical physics, and therefore could not be right. Thus he lectured in April 1908, at a mathematical congress in Rome. However, as it became clear that Planck's formula could no longer be ignored, Lorentz and Walther Nernst, among others, got the rich Belgian industrialist Ernest Solvay to fund an "urgently necessary" conference to reach agreement among scientists that the existing worldview of classical physics must not be attacked.

The "solution"—i.e. a foul compromise—was supplied by Niels Bohr with help of the young mathematical genius Heisenberg. The characteristic of this "matrix mechanics" (as Max Planck called it), was that real natural processes must be made to fit a well-functioning mathematics. The situation recalled the dilemma of the 16th Century, respecting the understanding of the motion of the heavenly bodies. Before Johannes Kepler's precise investigation of the orbit of Mars in his *Nova Astronomia*, and his discovery of the true law of motion (which implicitly contained within it the natural constant of gravitation), there was just confusion among the different "models," none of which had anything to do with the actual processes of Nature. Planck was conscious of the positivist and sophistic mindset, which always led into a deeper dilemma.

Later, as he became active in opposition to the Nazis, Planck noted Kepler's belief in "something *transeunt* over science," which drove him to say – in spite of the mathematically astonishingly correct results of the "models" of Ptolemy, Copernicus, and Brahe: all models are false, and I will find the truth:

"Can such a deeper conception of science be the basis for a guiding philosophy to life one's life by? We find the surest answer to this question by looking back in history to the men who embraced such a conception of science as their own, and for whom it indeed served this purpose. Among the numerous physicists, for whom their science helped them endure and glorify a miserable life, we remember … in the first rank … Johannes Kepler. Outwardly, he lived his life under beggarly conditions, disappointment, gnawing hunger, constant economic pressure…. What kept him alive and able to function through it all was his science, but not the numerical data of the astronomical observations in themselves, but his abiding faith in the power of a lawful intelligence in the universe. One sees how significant that is in a comparison with his employer and master Tycho Brahe. Brahe possessed the same scientific knowledge, the same observational data, yet he lacked the faith in the great eternal laws. Thus Tycho Brahe remained one among many worthy investigators, while Kepler was the creator of the new astronomy."

The mathematical "wunderkind" Heisenberg flunked the physics course under Professor Kirchoff, because he had no understanding of experimental physics. But in spite of this, he got powerful back-up from the Bohr faction for his development of Quantum Mathematics. This "solution" was given detailed philosophical justification through the "uncertainty principle" at the so-called "Bohr festivals" in Göttingen—as Bohr's chatty lectures were called.

Einstein: God Does Not Play Dice!

In 1894, Planck was admitted to the Prussian Academy of Sciences. Here he attempted to extend thermodynamics to other conditions, and thereby to delimit the Clausius entropy principle, as "it is completely unfounded, simply to assume that changes in Nature always proceed in the direction from lesser to greater probability." When Planck was chosen in 1912 alongside Wilhelm Waldeyer as one of the standing members of the physical-mathematical group in the Prussian Academy, and in 1913 as

Rector of Berlin University, he soon made an effort to bring Albert Einstein to Berlin as theoretical physicist, because he admired his work on Relativity Theory and. above all, his rigorous honesty on fundamental questions of natural knowledge. Planck's first official act consisted in the creation of a second chair of theoretical physics, which he offered to Einstein as a distinguished professor.

Symptomatic of the fundamental errors of the Bohr-Heisenberg type of "mathematical" analysis of Nature, which is, for all intents and purposes, a self-deception, is a discussion between Einstein and Heisenberg in the spring of 1926 in Berlin, after Heisenberg had presented his new mathematics for the first time at the University of Berlin. After the colloquium, Einstein asked Heisenberg for a fuller discussion, which Heisenberg later gave an account of in his Notes (pp. 92-95) "*Der Teil und das Ganze*" (The Part and the Whole):

"But as we were entering the apartment, he opened up the conversation at once with a question, which went straight to the philosophical assumptions of my research: `What you have just told us, is very exceptional. You assume that there are electrons in the atom, and there you certainly are correct. However, the paths of the electrons in the atom, —these you want to abolish completely, although one can still directly observe the electron tracks in a cloud chamber. Can you explain to me somewhat more precisely the reason for these remarkable assumptions?'

`The paths of the electrons in the atom cannot be observed,' I replied, `however the radiation, which is emitted from an atom during the process of relaxation, can be inferred directly from the frequency of oscillation and the associated amplitude of the atomic electron. In present-day physics, the complete knowledge of the frequency and amplitude serves as something like a surrogate for knowledge of the electron paths. But as it is still reasonable in a theory to assume only the magnitudes which can be observed, it seems to me natural to introduce these only, as representatives, so to speak, for the electron orbitals.'

`But you don't really believe that one can assume only observable quantities in a physical theory,' Einstein countered.

`I thought,' I asked amazed, `that you had directly applied such thoughts to the foundations of your relativity theory? You had stressed that one should not speak of absolute time, as one cannot observe this absolute time. Only the data of clocks, whether they be in a moving or stationary reference frame, are proper for the determination of time.'

`Perhaps I have made use of this type of philosophy,' answered Einstein, `but it is nonsense, nevertheless. Or, I can say more cautiously, that it may be of heuristic value to recall something which one actually observed. However, from a principled standpoint it is completely false to wish to base a theory only on observable magnitudes. Because, in reality, it is exactly the other way around. The theory first determines what one can observe .… I have the suspicion that you will later encounter difficulties in your theory exactly on this point of which we have just spoken. I want to motivate that more exactly. You pretend that you could just leave everything as it is, on the observational side of science, employing the language just as it has been used up to now, to describe what the physicists observe. However, if you do that, you must then also say: In the cloud chamber we observe the path of the electron in the chamber. However in the atom, there is no longer a path for the electron, in your opinion. But this is obviously absurd. Simply by making smaller the space in which the electron moves, the concept of a path cannot be annulled.' "

When Heisenberg then, obviously confusing mathematics with real Nature, argues that the great power of persuasion of his viewpoint emanates from "the simplicity and beauty of mathematical schema, which is suggested to us by Nature," Einstein nails him on the self-deception which is implied. As Heisenberg reports:

"`The experimental test," Einstein noted, `is certainly the trivial precondition for the correctness of a theory. However, one can never control and recheck everything. So, what you said about simplicity interests me even more. However, I would never claim to really understand what this simplicity of natural law is all about.' "

One must at least grant the very young and enthusiastic Heisenberg that he made the effort to get an honest understanding, mathematician that he was, in order to be able to grasp this paradox in its totality. Not until his later years was it clear to him that truth wore a different face.

Second World War: The End of Science?

In spite of very serious personal misfortunes (within just a few years Planck lost his younger son in the First World War, and both his twin daughters after the birth of their first child), he never relinquished his sense of responsibility for others, above all for the next generation, and, therefore, for the future of science. One can assert from the start, that, without him, the great breakthrough in nuclear physics achieved by his students Otto Hahn, Lise Meitner, and Fritz Strassmann would never have succeeded.

At the end of the First World War, the now 60-year-old Planck, positioned at the pinnacle of the Prussian Academy of Science, strove as hard as he could for the reconstruction of the scientific institution. Together with Prussian Minister of Culture Friedrich Schmidt-Ott and academy members Haber and von Harnack, he organized the *Notgemeinschaft der deutschen Wissenschaft* (Emergency Organization of German Science), in which scientists from all regions, professions, and political boundaries could join forces in order to obtain urgent financial means. After his retirement to emeritus status in 1926, Planck continued to work tirelessly through a very active lecture schedule, as editor of the *Annalen der Physik,* and in the founding of the Deutsches Museum in Munich. But the passage of years only brought more decay to the house of science: The economic crisis caused the income of the Emergency Organization to sink ever lower, while at the same time extremism and anti-Semitism spread within the academic establishment. Positions were filled only with Aryans, even when better qualified Jewish applicants were available. And, as with today's Greenies, Hitler and his followers took an increasingly negative attitude towards science and technology, and held them responsible for both overproduction and mass unemployment. After the takeover by the Nazi Party (NSDAP) in 1933, the situation became dangerous for many scientists, and leading figures like Einstein and Schrödinger had to leave the country. Incendiary flyers against Einstein. were distributed. Owing to the constant attacks against the alleged "Jewish quantum physics" or "Jewish relativity theory," the climate became unbearable, and the scientific landscape was turned into a desert.

Planck, too, was near the point of resigning his positions, and Heisenberg was considering emigration, but then, considering the gloomy prospects for the nation's future, they decided to fight on with the motto *In Deutschland bleiben, weiterarbeiten und retten* (To remain and keep working to save and free Germany). Together with his son Erwin, Planck was a member of the *Mittwochs-Gesellschaft* (Wednesday Club), which was broken up after the July 20, 1944 attempt on Hitler's life. Many members of the *Mittwochs-Gesellschaft* were found guilty of complicity and put to death on February 23, 1945, among them Planck's son Erwin and his childhood friend Ernst von Harnack.

For the 87-year-old Max Planck, the news of the deaths almost killed him, but he doggedly carried on putting priority on his public lectures, in order "to fulfill the desire of a struggling humanity for truth and knowledge, above all the youth." His life's motto was a famous saying from his adored Gottfried Wilhelm Leibniz: *"Sieh zu, was du tust; sag an, warum du es tust; denn die Zeit fliesst dahin"* (Watch what you do; say why you do it; for time races by). On October 4, 1947, Planck died at the age of 89, after multiple strokes. His legacy certainly remains very alive, and cries out to scientists: Do not cheat yourself of the truth, if only because theory is so beautifully simple and "the mind is so lazy," as Leibniz put it.